

# Phase Transition in Light-Front $\phi_{1+1}^4$

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## Abstract

We reproduce Chang's duality condition in a regularized  $\phi_{1+1}^4$  theory quantized on a light front. The regularization involves higher derivatives in the Lagrangian, renders the model finite in the ultraviolet, and does not require introduction of a finite size of the system. It is demonstrated that the light-front quantization is a natural way to treat systems with higher derivatives. The phase transition is related to the presence of tachyons in the regularized theory. Prospects for computing the critical coupling in this formulation are briefly discussed.

# 1 Introduction

Light-front quantization promises to become an alternative (to lattice formulation) computational approach to quantum field theories (for a review, see [1]). The common objection against it is that it predicts triviality of the vacuum, and, therefore, has trouble addressing the existence of phase transitions in field theories. A good test case to study this issue is the  $\phi_{1+1}^4$  model. For this model, the presence of a second order phase transition is proved rigorously [2], and demonstrated [3] with a duality relation derived between the couplings of the theories with different signs of the mass squared term in the Lagrangian. The duality condition implies the two theories with different signs on the mass squared term are identical. The duality relation maps the strong coupling limit of the theory with positive mass squared (i.e., with a single minimum of the potential) to the small coupling limit of the theory with a negative mass squared (i.e., with two minima of the potential). There is also a lattice computation for the value of the critical coupling [4].

If the light-front quantization is a viable scheme for nonperturbative computations, it should reproduce both the duality condition, and the numerical value of the critical coupling from the lattice computation. There are attempts in the literature within light-front quantization to achieve this (see [7, 8, 5, 6]).

Lately, the prevailing view maintains that the zero mode in light-front  $\phi_{1+1}^4$  theory accounts for the properties of the phase transition via its non-linear constraint (see, for example, [6]). In this "discretized light-cone quantization" or "DLCQ" approach, the system is put in a finite box along a light-like direction. According to one report [8], DLCQ is unable to reproduce the lattice results for the critical coupling, and yields a wrong value for the critical exponent. An alternative DLCQ investigation developing the framework of [9] is now under way, and has given promising results [11]).

Another objection is that the treatment of the zero modes is far from clearcut in that they are expressed in terms of the dynamical modes with a classical solution, and, subsequently, these expressions are treated as operators.

A more intuitive objection against the traditional form of DLCQ is that it seems to be at odds with the more conventional (equal-time and Euclidean lattice) approaches. For example, in the lattice computation of  $\phi_{1+1}^4$  theory, the presence of the phase transition is closely related

to the ultraviolet divergence present in the one divergent diagram in this model. The same relation to the divergent tadpole diagram shows up in the derivation of the duality relation due to Chang [3].

The relation of the ultraviolet divergence to the phase transition seems to be a fundamental feature, which one may expect to show up in any treatment. At present, there is no obvious connection between the zero mode DLCQ treatment of the phase transition and the ultraviolet divergence of the model. Furthermore, to our knowledge, there is no derivation of Chang's duality relation in the DLCQ treatment.

Reproducing Chang's duality condition constitutes a challenge for DLCQ because the ultraviolet divergence is independent of the mass of the excitation appearing in the spectrum at small coupling. The role of the mass in the DLCQ ultraviolet counterterm is played by the inverse size of the system along the light-like direction. But the phase transition emerges only in the limit of the infinite size of the system. We conclude that DLCQ regularization may not accomodate straightforwardly the phase transition.

An alternative approach is considered in [9], where the zero mode is discarded and a state of a soliton anti-soliton pair is constructed for the model with the negative sign of the mass term. Here the role of the infinite size limit in the emergence of the phase transition is stressed, but the role of the ultraviolet divergence is unclear, and there is no connection made with Chang's duality relation.

An approach sustaining a contact with the equal-time formulation is developed in [5]. In this paper, the light-front treatment of the phase transition is related to the ultraviolet divergence, the divergence is regularized, and Chang's duality relation is reproduced in light-front quantization. However, this approach differs from the DLCQ formulation, so one loses contact with the computational simplicity of DLCQ.

In this paper, we present a novel approach to light-front quantization of  $\phi_{1+1}^4$ . In a way, it continues the attitude of [5] that the regularization of the ultraviolet divergence of the theory is a crucial ingredient. In contrast to [5], we use a nonperturbative regularization introduced on the level of the Lagrangian of the model. One option is to use regularization with a light-front lattice [10]. We choose not to follow this option here, because it breaks conservation of light-front momentum. To retain the conservation of the light-front momentum seems to be crucial, since defining the state space in sectors of definite light-front momentum is the foundation of the computational capacity

of light front quantization. Instead, we chose to regularize the theory with higher derivatives and to conserve light-front momentum.

The light-front formulation is a natural way to quantize theories with higher derivatives because the number of light-front time derivatives is two times smaller than the corresponding number of the derivatives in the conventional physical time. Therefore, light-front quantization of a theory whose Lagrangian is quadratic in the Laplacian of the field is still a quantization of a theory whose equations of motion involve only up to second derivatives over the light-front time. The use of the light-front quantization to quantize the theories with higher derivatives is one of the key suggestions of this paper.

The quantization we present here leads to the presence of tachyons in the spectrum. Their mass parameter goes to infinity when the higher derivative terms are switched off (i.e., when the ultraviolet regularization is removed). For this reason, the tachyons decouple in the limit of the regularization removed. Still, they influence the dynamics of the real particles. In particular, if the tachyon dynamics implies a phase transition, the vacuum will become a tachyon condensate after the phase transition. Among the tachyons, there are excitations with small longitudinal momenta. In this way, the present approach does not deny the importance of the zero modes (i.e., the modes of zero light-front momentum) for the phase transition, but helps to consider them in a new way. We stress that there are no constraints in this light-front Hamiltonian treatment of the regularized theory, and there are no nondynamical degrees of freedom in this treatment.

This paper formulates our approach, reproduces Chang's duality condition, and discusses the prospects for transforming our approach into a computational scheme. In the next section, we discuss our choice of the ultraviolet regularization. In the third section, we give the regularized model the light-front Hamiltonian treatment, i.e., define the canonincal variables, the light-front Hamiltonian, and the light-front longitudinal momentum. In the fourth section, Chang's duality condition is reproduced in a way that mimics closely the original derivation due to Chang. In the last section, we discuss what is needed to transform our approach into a computational scheme.

## 2 The Ultraviolet Regularization

The Lagrangian [= density] of the theory is

$$\mathcal{L} = \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V(\phi), \quad (1)$$

where  $g^{\mu\nu}$  is the Lorentz metric tensor (its only nonzero components are  $g^{+-} = g^{-+} = 1$ ). Hereafter, the indexes after a coma in a subscript on a field denote the partial derivatives, e.g.,  $\phi_{,\mu} \equiv \partial_\mu \phi$ . The potential of the model is

$$V(\phi) = \frac{m^2}{2}\phi^2 + \frac{g}{4}\phi^4. \quad (2)$$

Perturbatively, there is a single divergent diagram in this model. This is the tadpole diagram that appears from a pairing of two fields involved in the same  $\phi^4$  vertex. Analytically, it is

$$T = \int \frac{dk_+ dk_-}{(2\pi)^2} \frac{i}{k^2 - m^2 + i\delta}, \quad (3)$$

where  $k^2 = 2k_+k_-$  is the Lorentz invariant momentum squared. The tadpole is divergent for two reasons. First, the propagator decays too slowly at infinite  $k^2$ . Second, the propagator is Lorentz invariant. To understand this, assume that a term  $(k^2)^2/M^2$  is introduced as a regulator in the denominator of (3). Go over to new integration variables,  $(k_+, k_-) \rightarrow (k^2, k_-)$ . Now the integral over  $k^2$  converges, but the integral over  $k_-$  is still divergent. The reason for this is the infinite volume of the group of Lorentz transformations. The extra divergence related to the infinite volume of the Lorentz group generally appears if a regularization breaks the conventional structure of the poles of Feynman integrands, and prevents the use of the Euclidean formulation. We conclude that the use of ultraviolet regularization with higher derivatives requires breaking of Lorentz invariance.<sup>1</sup>

Our choice for the regularized Lagrangian [= density] is

$$\mathcal{L}_r = \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + \frac{1}{2}\epsilon t^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + \frac{1}{2M^2}(\Box\phi)^2 - V(\phi), \quad (4)$$

where  $\Box\phi = g^{\mu\nu}\phi_{,\mu\nu}$ . The regularization is removed in the limit  $\epsilon \rightarrow 0, M \rightarrow \infty$ . As we will see, upon quantization,  $M$  becomes the mass parameter of the tachyons. The tensor  $t^{\mu\nu}$  is a symmetric tensor

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<sup>1</sup>We thank Prof. A. Vainshtein for discussing this point.

whose nonzero components in a Lorentz frame are  $t^{++} = 1, t^{--} = -1$ . The role of the term with the  $t$ -tensor is to break the Lorentz invariance. The signs of the components of this tensor are chosen to have a nonnegative light-front Hamiltonian (see below).

In the next section, we perform the light-front quantization of the model with the Lagrangian (4). This quantization leads to the field propagator of the following form:

$$\tilde{G}_r(k) = \frac{i}{k^2 - m^2 + \epsilon k_t^2 + (k^2)^2/M^2 + i\delta}, \quad (5)$$

where  $k_t^2 \equiv t^{\mu\nu} k_\mu k_\nu$ . With this propagator, the tadpole

$$T = \int \frac{dk_- dk_+}{(2\pi)^2} \tilde{G}_r(k)$$

is finite. Feynman diagrams not involving the tadpole have finite limits when the regularization is removed. These limits coincide with the corresponding diagrams of the original theory with the Lagrangian (1). From this we conclude that at least perturbatively the regularized theory reproduces the original theory at the limit the regularization is removed.

### 3 Light-Front Quantization of the Regularized Theory

Let us start the quantization procedure by writing down the expression for the density of the energy momentum tensor of the regularized theory. The Lagrangian [= density] (4) is expressed in terms of the field and derivatives of the field. The first and the second derivatives of the field are involved. Noether's procedure yields

$$(\theta_r)_\rho^\mu = \frac{\partial \mathcal{L}_r}{\partial \phi_{,\mu}} \phi_{,\rho} + \frac{\partial \mathcal{L}_r}{\partial \phi_{,\mu\nu}} \phi_{,\nu\rho} - \left( \partial_\nu \frac{\partial \mathcal{L}_r}{\partial \phi_{,\mu\nu}} \right) \phi_{,\rho} - \mathcal{L}_r \delta_\rho^\mu. \quad (6)$$

An integral of this density gives the light-front component of the total momentum of the system:

$$P_- = \int dx^- (\theta_r)_-^+ = \int dx^- [(\phi_{,-})^2 + (\epsilon \dot{\phi} - \frac{2}{M^2} \square \phi_{,-}) \phi_{,-}]. \quad (7)$$

The overdot above denotes the derivative over  $x^+$ , which is considered as dynamical time, and the line  $x^-$  taken at a fixed value of the light-front time is the manifold where the initial condition for the field is

set. Similarly, for the momentum component along the plus direction, we obtain

$$P_+ = \int dx^- (\theta_r)_+^+ = \int dx^- \left[ \frac{\epsilon}{2} (\dot{\phi}^2 + (\phi_-)^2) + \frac{1}{2M^2} (\square\phi)^2 + V(\phi) \right]. \quad (8)$$

The canonical coordinates of the regularized model are the values  $\phi(x^-)$  (their dependence on  $x^+$  describes the dynamics). The conjugated momenta are obtained as the variational derivatives of the Lagrangian [= in  $\dot{\phi}$ ] (4) over  $\dot{\phi}$ :

$$\pi = \phi_{,-} + \epsilon \dot{\phi} - \frac{2}{M^2} \square\phi_{,-}. \quad (9)$$

The above components of the total momentum expressed in terms of the canonical variables are

$$P_- = \int dx^- \pi \partial_- \phi, \quad (10)$$

$$P_+ = \int dx^- \left[ \frac{1}{2} (\pi - \phi_{,-}) \frac{1}{\epsilon - 4\partial_-^2/M^2} (\pi - \phi_{,-}) + \frac{\epsilon}{2} \phi_{,-}^2 + V(\phi) \right]. \quad (11)$$

The expression for  $P_-$  coincides with the one for the momentum space component in the equal-time quantization (with the natural replacement of the integral over the space by the integral over the light-front). We use this observation to diagonalize the momentum  $P_-$ . It is achieved with the following decomposition of the field and the conjugated momentum:

$$\phi(x^-) = \int \frac{dl}{\gamma_m(l)\sqrt{4\pi}} [a_l \exp(-ilx^-) + a_l^\dagger \exp(ilx^-)], \quad (12)$$

$$\pi(x^-) = \int \frac{dl}{i\sqrt{4\pi}} \gamma_m(l) [a_l \exp(-ilx^-) - a_l^\dagger \exp(ilx^-)]. \quad (13)$$

Here  $l$  has the meaning of the longitudinal momentum of the excitation (see below), and ranges from negative to positive infinity. The  $\gamma_m(l)$  is an arbitrary real even function of the longitudinal momentum  $l$  at this stage. We will specify it when we diagonalize the free light-front Hamiltonian. Notice that the quantization proceeds in complete analogy with the equal-time quantization. As in the equal-time quantization, it is possible to diagonalize the free Hamiltonian (the role of

the Hamiltonian is played in the light-front quantization by  $P_+$ ) by fitting the dependence of  $\gamma_m(l)$  on  $l$ . As we will see in a moment, this dependence will be different at different masses. This is why we put the subscript  $m$  on  $\gamma_m(l)$ . Turning back to  $P_-$ , in terms of the creation-annihilation operators introduced above, it is

$$P_- = \int dl l a_l^\dagger a_l. \quad (14)$$

We see that  $a_l^\dagger$  creates the excitation whose momentum component along the direction  $x^-$  is  $l$ , and the negative values of  $l$  are not forbidden. At the same time,  $P_+$  defined in (11) is explicitly positive. Thus, the excitations with negative  $l$  are tachyons (i.e., their mass squared,  $M^2 = 2P_-P_+$ , is negative).

Now let us diagonalize the free Hamiltonian. To this end, put  $V(\phi) = m^2\phi^2/2$  into (11), substitute the expansions of the field and canonical momentum over the creation-annihilation operators, and require that the coefficient of the combination  $(a_l a_{-l} + a_l^\dagger a_{-l}^\dagger)$  vanishes. This yields the expression for  $\gamma_m(l)$ :

$$\gamma_m(l) = [l^2 + (m^2 + \epsilon l^2)(\epsilon + 4l^2/M^2)]^{\frac{1}{4}}. \quad (15)$$

Again notice that the equal-time quantization formally differs only by this expression (in the equal-time,  $\gamma_m(l)$  is replaced with  $\sqrt{\omega_m(l)} = [l^2 + m^2]^{1/4}$ ).

At the above choice of  $\gamma_m(l)$ , the free light-front Hamiltonian is as follows:

$$P_+ = \int dl \nu_m(l) a_l^\dagger a_l, \quad (16)$$

where the light-front energy of the excitation with the light-front momentum  $l$  is

$$\nu_m(l) = \frac{\gamma_m^2(l) - l}{\epsilon + 4l^2/M^2}. \quad (17)$$

There is a qualitative difference here with the equal-time quantization. In the equal-time quantization, the energy of the excitations is an even function of  $l$ . In the light-front quantization, the excitations with positive light-front momentum are qualitatively different from the excitations with negative light-front momentum. The former are the particles of mass  $m$ , the latter are the tachyons of mass parameter  $M$ .



To see this, consider  $\nu_m(l)$  in the limit of regularization removed,  $\epsilon \rightarrow 0, M^2 \rightarrow \infty$ . It has the meaning of the momentum component along the  $x^+$  direction of the excitation whose momentum component along the  $x^-$  direction is  $l$ . Therefore, we expect that the product  $2l\nu_m(l)$  is independent of  $l$  in this limit (because it has the meaning of the mass squared of the excitation). Otherwise, the Lorentz invariance is violated even in the limit of the regularization removed. Formally, to get the expected dependence on  $l$  in  $\nu_m(l)$  after the removal of the regularization at negative  $l$ , we need to neglect  $\epsilon$  with respect to  $4l^2/M^2$  in the denominator of (17). In other words, we should take the limit  $\epsilon \rightarrow 0, M \rightarrow \infty$  in such a way that  $\epsilon M^2 \rightarrow 0$ . We also should keep the momentum  $l$  nonzero. After all these reservations, in the limit of the regularization removed, we have

$$\nu_m(l) \rightarrow \theta(l) \frac{m^2}{2l} - \theta(-l) \frac{M^2}{2l}, \quad (18)$$

where  $\theta(l)$  vanishes at negative  $l$  and equals the unit at positive  $l$ .

Note that the above limit does not hold at  $l = 0$ :

$$\nu_m(0) = \frac{m}{\sqrt{\epsilon}}. \quad (19)$$

We see that in the regularized theory the mode of zero longitudinal momentum breaks Lorentz invariance. Evidently, this mode should be treated separately. We conjecture that this mode can be neglected at small coupling, and causes the phase transition at the critical coupling. We further discuss this point in the last section of the paper.

We close this section with a derivation of the propagator of the free field in the regularized theory. The propagator is defined as

$$G_r(x) = \theta(x^+) \langle \phi(x) \phi(0) \rangle + \theta(-x^+) \langle \phi(0) \phi(x) \rangle. \quad (20)$$

Here the time dependence of the field is obtained from (12) with the replacements

$$\exp [\pm i l x^-] \rightarrow \exp [\pm (i \nu_m(l) x^+ + l x^-)].$$

It is easy to check that this time dependence is in agreement with the equations of motion for the field implied by the regularized Lagrangian (4) at  $V(\phi) = m^2 \phi^2/2$ . With the above field decomposition, the propagator's Fourier transform  $\tilde{G}(k) \equiv \int d^2 x \exp(-i k x) G(x)$  is

$$\tilde{G}(k) = \frac{1}{2i\gamma_m^2(k_-)} \left[ \frac{1}{\nu_m(-k_-) + k_+ - i\delta} + \frac{1}{\nu_m(k_-) - k_+ - i\delta} \right], \quad (21)$$

where  $k = (k_+, k_-)$  is a two-dimensional vector. It is easy to check that this expression does coincide with the propagator (5).

## 4 Chang's Duality Condition

Let us recall the derivation of Chang's duality condition [3]. If  $V(\phi) = -\mu^2\phi^2/4 + g\phi^4/4$  in (1), the small coupling limit at negative mass squared in the Lagrangian cannot be treated with the same field decomposition into the creation-annihilation operators as it can be at the positive mass squared term. This is the case because, at the negative mass squared term, it is not possible to remove from the free Hamiltonian the term  $a_l a_{-l} + a_l^\dagger a_{-l}^\dagger$  with a choice of the real coefficient in the field's decomposition.

To treat the small coupling limit at negative mass squared, one should shift the field by a value that minimizes the potential (there are two symmetric minima in this case). The fluctuations of the field around the extremal value are then expanded as in the case of the positive mass term, but with the new mass parameter  $\mu$ . The new mass parameter is defined from the second derivative of the potential in the minimum. The Hamiltonian is then defined as a normal ordering with respect to the creation-annihilation operators participating in the mode expansion of the fluctuations around the minimum.

Chang compares the Hamiltonian obtained in this way with the Hamiltonian obtained for the model with the positive mass squared term by the normal ordering with respect to the creation-annihilation operators related to the mass parameter from the Lagrangian. To make the comparison, he first considers the changes induced in the Hamiltonian by the change in the ordering prescription caused by switching over from mass  $m$  to mass  $\mu$  in the field decomposition. Next he notices that these changes transform the Hamiltonian to the one obtained for the case with the negative mass term if the following duality condition is satisfied:

$$\frac{m^2}{g} + \frac{3}{4\pi} \ln \frac{m^2}{g} = \frac{3}{4\pi} \ln \frac{\mu^2}{g} - \frac{\mu^2}{g}. \quad (22)$$

This can be interpreted as a relation between dimensionless couplings  $f \equiv g/m^2$  and  $f_1 \equiv g/\mu^2$ . When  $f$  is large, there exists a small  $f_1$  such that the duality condition (22) is satisfied. The conclusion is that the theory with a positive mass squared term and large coupling

is equivalent to the theory with negative mass term (and nonzero vacuum expectation of the field) and a small coupling.

Chang's derivation rests on the relation between normal orderings of the products of the field operator with respect to different values of the mass parameter. We can repeat the reasoning of Chang for the light-front quantization of the previous section. The only change is in the formula relating the two ordering prescriptions. The formula Chang uses to relate two normal orderings is

$$N_m(e^{i\beta\phi}) = (\frac{\mu^2}{m^2})^{\beta/8\pi} N_\mu(e^{i\beta\phi}). \quad (23)$$

The factor by the normal product in the rhs,  $F_{et}(\beta; \mu, m) = (\frac{\mu^2}{m^2})^{\beta/8\pi}$  (the subscript "et" on the factor indicates that this is the equal-time quantization), is related to the factor in the field decomposition as follows:

$$F_{et}(\beta; \mu, m) = \exp \left[ \frac{1}{2} \beta (\Delta_{et}(\mu) - \Delta_{et}(m)) \right], \quad (24)$$

where

$$\Delta_{et}(m) = \int \frac{dl}{4\pi\omega_m(l)}.$$

Here  $\omega_m(l)$  is involved in the equal-time field decomposition:

$$\phi_{et}(x^1) = \int \frac{dl}{\sqrt{4\pi\omega_m(l)}} [a_l \exp(-ilx^1) + a_l^\dagger \exp(ilx^1)],$$

where  $x^1$  is the space coordinate. We can repeat all the reasoning for the light-front quantization. The normal orderings are related in this case by the formula

$$N_m(e^{i\beta\phi}) = F_{lf}(\beta; \mu, m) N_\mu(e^{i\beta\phi}), \quad (25)$$

where

$$F_{lf}(\beta; \mu, m) = \exp \left[ \frac{1}{2} \beta (\Delta_{lf}(\mu) - \Delta_{lf}(m)) \right]. \quad (26)$$

Here

$$\Delta_{lf}(m) = \int \frac{dl}{4\pi\gamma_m^2(l)}.$$

This is implied by the comparison of the light-front field decomposition in (12), and the above field decomposition of the equal-time quantization. We see from this comparison that the light-front formulas for normal ordering are retained from the corresponding equal-time formulas with the replacement  $\omega_m(l) \rightarrow \gamma_m^2(l)$ .

We now use the explicit expression (15), and check that

$$F_{lf}(\beta; \mu, m) \rightarrow F_{et}(\beta; \mu, m)$$

in the limit of the regularization removed. We conclude that the relation in the light-front quantization between the normal orderings corresponding to different masses coincides with the one obtained in the equal-time quantization.

From this we conclude that Chang's duality relation is valid for the light-front Hamiltonian of the previous section.

## 5 Discussion

As we have seen in the previous section, the light-front quantization of the theory of Eq. (4) is equivalent at large coupling to the light-front quantization of a theory obtained from Eq. (4) with a changed sign of the mass squared term. The latter theory should be taken at a small coupling. It has a nonzero vacuum expectation of the field. On the other hand, at small coupling, the theory of Eq. (4) with a positive mass squared term has zero vacuum expectation of the field. The conclusion is that somewhere on the way from small coupling to large coupling there is a phase transition. We recall that we reached this conclusion for the light-front quantization of the theory whose Lagrangian is given in Eq. (4).

At small coupling, there exist tachyons in the spectrum. The mass parameter of the tachyons goes to infinity as the regularization is removed. To make the above approach a computational scheme, we should study the decoupling of tachyons. In perturbation theory, tachyons decouple.

A separate treatment should be given to the modes whose longitudinal momentum is of the order  $\sqrt{\epsilon}$  ( $\epsilon$  is the regulator breaking Lorentz invariance in our approach). Perturbatively, these modes give a finite contribution to the energy density of the vacuum in the limit of the regularization removed (despite the fact that their light-front energy goes to infinity in this limit). Therefore, they are the most viable candidates for the role of the modes responsible for the phase transition. The modes of small longitudinal momentum should be integrated out under certain assumptions on their nonperturbative dynamics (the latter may still be out of reach for an analytical treatment). In this way,

the dynamics of conventional modes will be parameterized by an effective Hamiltonian. It requires further study if this kind of approach will be able to become a computational scheme.

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